CS557 W09

# Homework #8

Due Tuesday, 29 September 2009

1. A cubic B-spline curve has a knot vector

$$[0, 0, 0, 1, 3, 3, 3]$$

   and control points $\mathbf{P}_0 = (0, 0)$, $\mathbf{P}_1 = (4, 4)$, $\mathbf{P}_2 = (8, 0)$, $\mathbf{P}_3 = (12, 4)$, and $\mathbf{P}_4 = (0, 8)$. Insert a second knot at $t = 1$. What are the control point coordinates and their polar labels after the knot is inserted?

2. A cubic B-spline curve has a knot vector

$$[0, 0, 0, 1, 3, 3, 3]$$

   and control points $\mathbf{P}_0 = (0, 0)$, $\mathbf{P}_1 = (4, 4)$, $\mathbf{P}_2 = (8, 0)$, $\mathbf{P}_3 = (12, 4)$, and $\mathbf{P}_4 = (0, 8)$. Insert a knot at $t = 2$. What are the control point coordinates and their polar labels after the knot is inserted?

3. A cubic B-spline curve has a knot vector

$$[0, 0, 0, 1, 2, 2, 2]$$

   and control points $\mathbf{P}_0 = (0, 0)$, $\mathbf{P}_1 = (6, 12)$, $\mathbf{P}_2 = (12, 18)$, $\mathbf{P}_3 = (18, 6)$, and $\mathbf{P}_4 = (18, 0)$. What are the control points of the two Bézier curves that make up this B-spline? What is the second derivative of those two Bézier curves at the point where they meet?

4. A degree $n$ B-spline with $m$ control points in general position has

   _______ curve segments,

   _______ knot values in the knot vector

   _______ order continuity at single knots.

5. Given two cubic Bézier curves, $\mathbf{P}_{[0,1]}(t)$ and $\mathbf{Q}_{[1,2]}(t)$. $\mathbf{P}_{[0,1]}(t)$ has control points

$$\mathbf{P}_0 = (0, 0), \quad \mathbf{P}_1 = (1, 2), \quad \mathbf{P}_2 = (2, 2), \quad \mathbf{P}_3 = (3, 2)$$

   and $\mathbf{Q}_{[1,2]}(t)$ has control points

$$\mathbf{Q}_0 = (3, 2), \quad \mathbf{Q}_1 = (4, 2), \quad \mathbf{Q}_2 = (5, 2), \quad \mathbf{Q}_3 = (6, 4).$$

   Express these two Bézier curves in B-spline form using as few B-spline control points and knots as possible. Indicate the knot vector and the control points.

A recommended way to do this is to express the two Bézier curves as a single B-Spline with knot vector [0,0,0,1,1,1,2,2,2] and with control points

$$\mathbf{P}_0 = (0,0), \quad \mathbf{P}_1 = (1,2), \quad \mathbf{P}_2 = (2,2), \quad \mathbf{P}_3 = (3,2) \quad \mathbf{P}_4 = (4,2), \quad \mathbf{P}_5 = (5,2), \quad \mathbf{P}_6 = (6,4)$$

and then do "knot removal" to remove the knot 1 as many times as possible. Knot removal is the reverse of knot insertion.