CS 557
Homework #2
Due Tuesday, 8 September 2009

1. A cubic Bézier curve $\mathbf{P}_{[0,1]}(t)$ has control points $\mathbf{P}_0 = (0,0)$, $\mathbf{P}_1 = (54,108)$, $\mathbf{P}_2 = (135,135)$, and $\mathbf{P}_3 = (243,81)$. Find the (x,y) coordinates of the point $\mathbf{P}(\frac{1}{3})$ by evaluating the equation for a Bézier curve

$$\mathbf{P}(t) = \sum_{i=0}^{n} B_i^n(t)\mathbf{P}_i$$

where $B_i^n(t) = \binom{n}{i}(1-t)^{n-i}t^i$, $\binom{n}{i} = \frac{n!}{i!(n-i)!}$ and $n = 3$ in this case.

2. Subdivide the curve in Problem 1 at $t = \frac{1}{3}$. Sketch the control polygons for $\mathbf{P}_{[0,\frac{1}{3}]}(t)$ and $\mathbf{P}_{[\frac{1}{3},1]}(t)$, and write down the control point coordinates.

3. Express $\mathbf{P}_{[0,1]}(t)$ as a degree 4 Bézier curve (i.e., degree elevate the curve and give the Cartesian coordinates of its control points).

4. Express $\mathbf{P}_{[0,1]}(t)$ as a degree 2 Bézier curve (give the Cartesian coordinates of the control points of an equivalent degree 2 Bézier curve).

5. Find the hodograph of $\mathbf{P}_{[0,\frac{1}{3}]}(t)$; give the Cartesian coordinates of its control points.

6. Compute the second, third, and fourth derivatives of $\mathbf{P}_{[0,\frac{1}{3}]}(t)$, each evaluated at $t = \frac{1}{3}$. Find the first derivative using the hodograph; the second derivative using the hodograph of the hodograph, etc.

7. Compute the first, second, third, and fourth derivative of $\mathbf{P}_{[\frac{1}{3},1]}(t)$, each evaluated at $t = \frac{1}{3}$.

8. Write the parametric equation of the curve in Problem 1 using power basis polynomials:

$$x(t) = x_0 + x_1 t + x_2 t^2 + x_3 t^3$$

$$y(t) = y_0 + y_1 t + y_2 t^2 + y_3 t^3$$

9. Using the power-basis polynomials in 8), compute $x'(\frac{1}{3})$, $x''(\frac{1}{3})$, $x'''(\frac{1}{3})$, and $x^{(4)}(\frac{1}{3})$.

Hand in this homework in class on 8 September.