CS 557
Homework 11
Due Thursday, 8 October 2009

This homework set deals with polynomials in Bernstein form. We will use the notation

$$\langle f_0, f_1, \ldots, f_n\rangle^n(t) = \sum_{i=0}^{n} f_i B_i^n(t).$$

1. Find the product of
$$\langle 1,3,2\rangle^2(t) \quad \text{and} \quad \langle 4,1,2\rangle^2(t)$$

2. Multiple degree elevation can be performed by multiplying by a Bernstein polynomial whose coefficients are all one. Degree elevate $\langle 1,3,2\rangle^2(t)$ three times by multiplying it by $\langle 1,1,1,1\rangle^3(t)$.

3. "Deflate" the polynomial $\langle 0,3,4,9,4\rangle^4(t)$. That is, divide out the root at $t = 0$ and report the coefficients of the resulting degree-three polynomial.

4. Applying the Convex Hull Marching root finding algorithm to $\langle -1,2,6,1,-2\rangle^4(t)$, compute t_1.

5. Applying the Convex Hull Marching root finding algorithm to $\langle -1,1,1\rangle^2(t)$, compute t_1 and t_2.

6. The points of intersection between a rational Bézier curve
$$\mathbf{P}(t) = \frac{\sum_{i=0}^{n} w_i \mathbf{P}_i B_i^n(t)}{\sum_{i=0}^{n} w_i B_i^n(t)}$$
and a line
$$ax + by + cw = 0$$
can be computed by substituting the parametric equation of the curve into the implicit equation of the line, producing a polynomial in Bernstein form
$$f(t) = \sum_{i=0}^{n} f_i B_i^n(t), \qquad f_i = a x_i w_i + b y_i w_i + c w_i.$$
Find the polynomial $f(t)$ for the cubic Bézier curve with control points and weights:
$$\mathbf{P}_0 = (1,2),\ w_0 = 1, \quad \mathbf{P}_1 = (2,5),\ w_1 = 2, \quad \mathbf{P}_2 = (5,5),\ w_2 = 3, \quad \mathbf{P}_3 = (6,1),\ w_3 = 1$$
and the line
$$3x - y - 2 = 0.$$