**Instructor: Dr. Thomas W. Sederberg**, 3318 TMCB
14–16 February 2002
Closed book, no time limit.
Calculator allowed.
One $8\frac{1}{2} \times 11$ page of notes (one side of the sheet) allowed.
**Give complete answers and show all work!**

1. (16 points) The Smith curve curve is given by

$$\mathbf{P}(t) = \frac{t^2 - 2t + 1}{t^2 + 1}\mathbf{P}_0 + \frac{2t - 2t^2}{t^2 + 1}\mathbf{P}_1 + \frac{2t^2}{t^2 + 1}\mathbf{P}_2$$

a. **YES** **NO** Does this curve interpolate the endpoints? Why or why not?

b. **YES** **NO** Is this curve symmetric? Why or why not?

c. **YES** **NO** Is this curve coordinate system independent? Why or why not?

d. **YES** **NO** Does this curve obey the convex hull property? Why or why not?

2. (10 points) Give the equivalent power-basis polynomial equations $x(t)$, $y(t)$ for the cubic Bézier curve with control points:

$$\mathbf{P}_0 = (1, 1), \quad \mathbf{P}_1 = (3, 5), \quad \mathbf{P}_2 = (5, 6), \quad \mathbf{P}_3 = (7, 4).$$

Answer: $x(t) =$ ______________________, $y(t) =$ ______________________.

3. (10 points) A degree one NURBS curve has knot vector $[1, 3, 5, 6]$ and control points and weights:

$$\mathbf{P}_0 = (1, 1); w_0 = 1. \quad \mathbf{P}_1 = (3, 3); w_1 = 3. \quad \mathbf{P}_2 = (11, 7); w_2 = 1; \quad \mathbf{P}_3 = (10, 2); w_3 = 4.$$

Insert a knot at $t = 4$. Your answer should state the new knot vector and the Cartesian coordinates of the new control points.

4. (10 points) For a certain cubic polynomial $f(t)$, we have:

$$f(1) = 1; \quad f(2) = 2; \quad f(3) = 4; \quad f(5) = 15.$$

Solve for $f(4)$ using forward differencing.

5. (10 points) Recall that the equation for the Timmer PC curve is:

$$\mathbf{Q}(t) = (-2t^3 + 5t^2 - 4t + 1)\mathbf{Q}_0 + (4t^3 - 8t^2 + 4t)\mathbf{Q}_1 + (-4t^3 + 4t^2)\mathbf{Q}_2 + (2t^3 - t^2)\mathbf{Q}_3.$$

Given a Timmer curve with control points $\mathbf{Q}_0$, $\mathbf{Q}_1$, $\mathbf{Q}_2$, $\mathbf{Q}_3$, find the control points $\mathbf{P}_0$, $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$ of an equivalent cubic Bézier curve $\mathbf{P}(t)$. (Hint: Note that if $\mathbf{P}(t) \equiv \mathbf{Q}(t)$, then $\mathbf{P}(0) = \mathbf{Q}(0)$, $\mathbf{P}(1) = \mathbf{Q}(1)$, $\mathbf{P}'(0) = \mathbf{Q}'(0)$, and $\mathbf{P}'(1) = \mathbf{Q}'(1)$.)

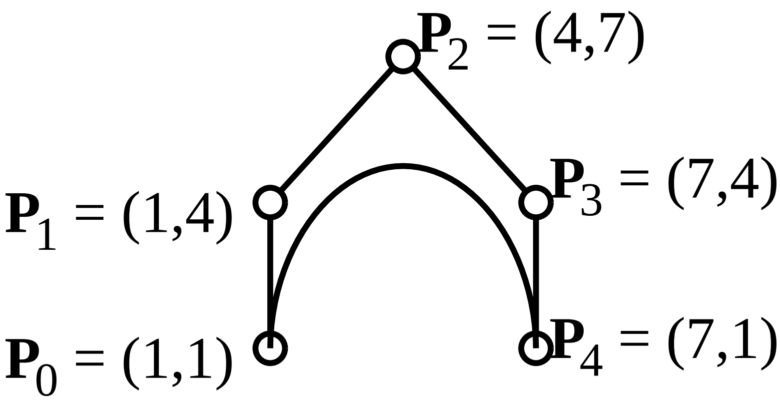
6. (10 points) A rational quadratic Bézier curve has control points and weights:

$$\mathbf{P}_0 = (0,0),\ w_0 = 1; \quad \mathbf{P}_1 = (4,3),\ w_1 = 2; \quad \mathbf{P}_2 = (0,5),\ w_2 = 4.$$

a. What is the curvature at $t = 0$?

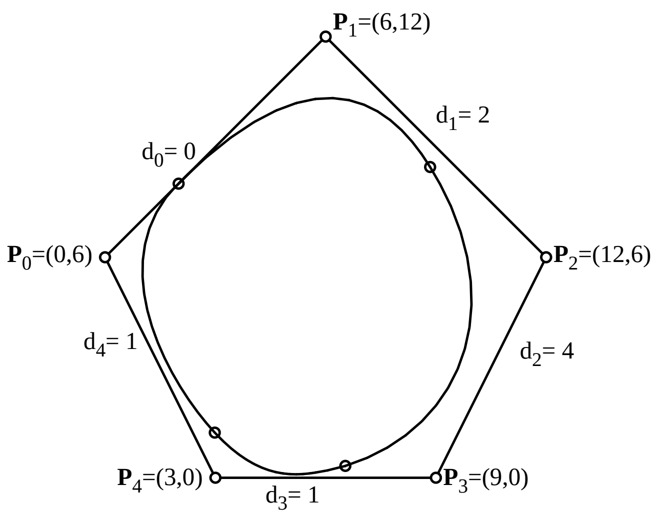
b. Is this curve a hyperbola, a parabola, or an ellipse, and why?

7. (10 points) This degree four polynomial Bézier curve begins at $t = 1$ and ends at $t = 5$, and is therefore also a B-spline with knot vector [11115555]. Insert a knot at $t = 3$. State the control point coordinates and knot vector after this knot insertion is performed.




8. (6 points) A degree five B-spline has a knot vector [a b c d e f g h i j k l m n o]. None of the knots are multiple.
How many Bézier curves comprise this B-spline?
What are the parameter domains of those Bézier curves?
If the control point (c d e f g) is moved, which of those Bézier curves are changed?

9. (8 points) This figure shows a periodic cubic B-spline whose edges are labelled with knot intervals. Insert a knot by splitting knot interval $d_2$ in half. Give the Cartesian coordinates of the new control points and label the edges of the resulting control polygon with the appropriate knot interval widths.




10. (10 points)

**True** **False** Any curve that is variation diminishing obeys the convex hull property.
**True** **False** Any Bézier curve can be represented as a B-spline curve.
**True** **False** According to Bezout's theorem, any two distinct circles intersect four times.
**True** **False** The hodograph of a rational cubic Bézier curve is a rational quadratic Bézier curve.
**True** **False** If you reparametrize a curve, the curvature at each point on the curve is unchanged.