Name: ____________________

CS 557
Sample Exam 1

Exam 1 will cover chapters 1–5. It will be closed book. You will be allowed one $8\frac{1}{2} \times 11$ page of notes (one side of the sheet).

These are some sample problems taken from exams in previous years.

1. (25 points) The Bizet curve is given by $\mathbf{P}(t) = (1-t)^3\mathbf{P}_0 + 2t(1-t)\mathbf{P}_1 + t(1-t)\mathbf{P}_2 + t^3\mathbf{P}_3$.

a. **YES** **NO** Does this curve interpolate the endpoints? Why or why not?

b. **YES** **NO** Is this curve symmetric? Why or why not?

c. What is the derivative vector at t=0 and t=1?

$P'(0) =$ ____________________ $P'(1) =$ ____________________

d. **YES** **NO** Is this curve coordinate system independent? Why or why not?

e. **YES** **NO** Does this curve obey the convex hull property?

f. Find the maximum distance between a Bizet curve with control points

$$\mathbf{P}_0 = (3,0); \quad \mathbf{P}_1 = (0,5); \quad \mathbf{P}_2 = (15,5); \quad \mathbf{P}_3 = (15,0)$$

and a Bizet curve with control points

$$\mathbf{P}_0 = (0,4); \quad \mathbf{P}_1 = (4,0); \quad \mathbf{P}_2 = (7,15); \quad \mathbf{P}_3 = (15,0)$$

2. (10 points) Convert the power basis rational curve

$$x = \frac{2t^2}{t^2+1} \qquad y = \frac{(t+1)^2}{t^2+1}$$

to rational Bézier form. That is, find the control points and weights for the equivalent rational Bézier curve.

Answer: $\mathbf{P}_0 = (\quad , \quad) w_0 = \quad ; \mathbf{P}_1 = (\quad , \quad) w_1 = \quad ; \mathbf{P}_2 = (\quad , \quad) w_2 = \quad .$

3. (10 points) Find the control points of the cubic Bézier curve for which

$$\mathbf{P}(0) = (0,0); \quad \mathbf{P}' = (6,6); \quad \mathbf{P}'' = (0,-12); \quad \mathbf{P}''' = (-6,6).$$

4. (5 points) At how many points do three arbitrary circles intersect? Why? (By intersect, we mean all three circles meet at the same point).

5. (15 points) Find the cubic blending functions for a curve

$$\mathbf{P}(t) = \mathbf{P}_0 b_0(t) + \mathbf{P}_1 b_1(t) + \mathbf{P}_2 b_2(t)$$

such that

$$\mathbf{P}(0) = \mathbf{P}_0; \quad \mathbf{P}(\frac{1}{2}) = \mathbf{P}_1; \quad \mathbf{P}(1) = \mathbf{P}_2; \quad \mathbf{P}'(\frac{1}{2}) = \mathbf{P}_2 - \mathbf{P}_0.$$

6. (10 points) Find the control points of the B-spline with knot vector

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given the following polar values

$$f(0,0,0) = (0,0); \quad f(0,0,1) = (0,4); \quad f(0,1,1) = (4,8);$$
$$f(1,1,1) = (8,8); \quad f(2,2,2) = (12,4).$$

7. (5 points) A degree four B-spline has a knot vector [a b c d e f g h i j k l m n o p q r s]. None of the knots are multiple. If the control point (c d e f) is moved, what are the parameter ranges of the underlying Bézier curves that are changed?

8. (10 points) Convert to degree four Bernstein basis the polynomial

$$t^3 + 1.$$

Answer:

$\_\_\_\_\_ B_0^4(t) + \_\_\_\_\_ B_1^4(t) + \_\_\_\_\_ B_2^4(t) + \_\_\_\_\_ B_3^4(t) + \_\_\_\_\_ B_4^4(t).$

9. (15 points) Find the control points $\mathbf{P}_1$ and $\mathbf{P}_2$ of a polynomial cubic Bézier curve which is curvature continuous with the circles of radius 2 and 4 as shown.

10. The equation of a degree 2 planar rational curve

$$x = \frac{t^2 - 1}{t^2 + 1}; \qquad y = \frac{2t}{t^2 + 1}$$

is substituted into the implicit equation of a second degree 2 curve $f(x, y) = 0$ yielding

$$t^3 - 2t^2 = 0.$$

What are the (x, y) coordinates of the points at which the two curves intersect?