Chapter 14

Free-Form Deformation (FFD)

Free-form deformation (FFD) is a technique for manipulating any shape in a free-form manner. Pierre Bézier used this idea to manipulate large numbers of control points for Béziersurface patches [B́74, B́78], and the power of FFD as a modeling tool was more fully explored in [SP86b]. This chapter discusses the 2D case of FFD.

A mathematician would say that 2D FFD is a map from $R^2 \rightarrow R^2$; that is, it defines a new position for every point in a given (normally rectangular) region. Any lines or curves that lie in that region are thus altered. In Figure 14.1, the FFD is adjusted using the nine control points. The undeformed scene appears at the left, and the right shows what happens the the grid and circle after the control points are moved. The grid is drawn here to help visualize how FFD works; in general, anything drawn inside of the initial, undeformed rectangle will experience the distortion, such as the text in Figure 14.2.

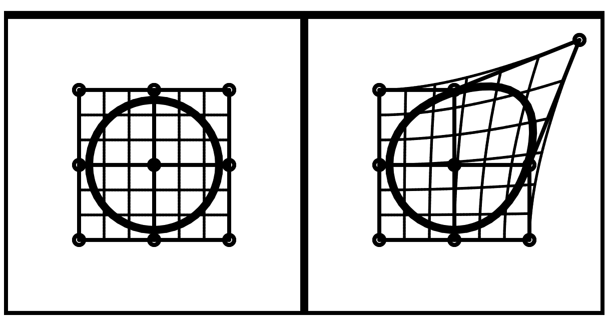

Figure 14.1: FFD example

If a shape is only partially inside the FFD region, one can control the degree of continuity between the deformed and undeformed portions of the shape by "freezing" rows of control points, as shown in Figure 14.3.

Denote by (X_{min}, Y_{min}) and (X_{max}, Y_{max}) the corners of a deformation region, and by m and n the degrees of the FFD function (there are to be $m+1$ vertical columns and $n+1$ horizontal rows of control points).

The deformation is defined in terms of a **rational bivariate tensor product Bernstein poly-**

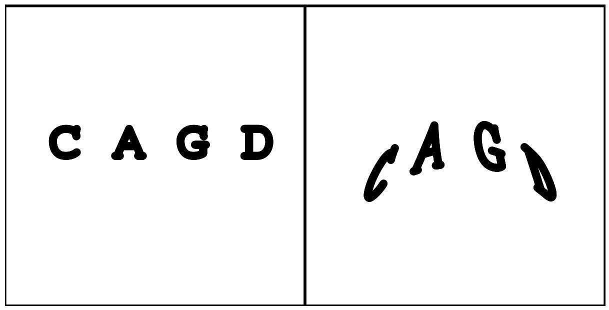

Figure 14.2: FFD example

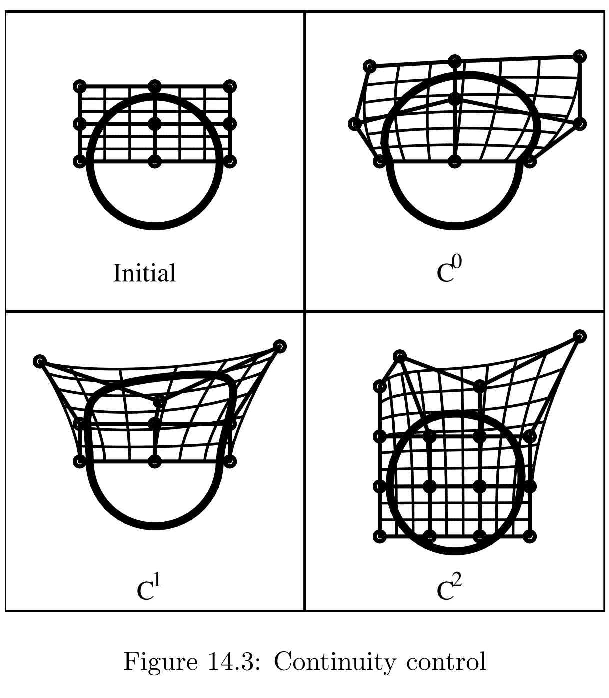

Figure 14.3: Continuity control

nomial which takes the form

$$\mathbf{X}(s,t) = \frac{\sum_{j=0}^{n}\sum_{i=0}^{m} w_{ij} B_i^m(s) B_j^n(t) \mathbf{P}_{ij}}{\sum_{j=0}^{n}\sum_{i=0}^{m} w_{ij} B_i^m(s) B_j^n(t)} \qquad (14.1)$$

where $B_i^n(t)$ and $B_j^m(s)$ are Bézier blending functions, and s and t are the local coordinates of a point with respect to the deformation region.

The s and t coordinates of a point in the deformation region range between 0 and 1 (see Figure 14.4). Thus, for a point (x, y) within the rectangular region,

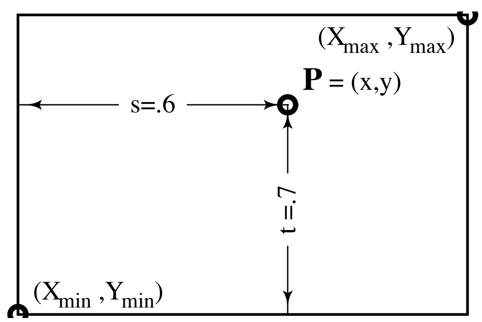

Figure 14.4: FFD local coordinates

$$s = \frac{x - X_{min}}{X_{max} - X_{min}}, \qquad t = \frac{y - Y_{min}}{Y_{max} - Y_{min}}. \qquad (14.2)$$

The control point values $\mathbf{P}_{ij}$ are the actual (x, y) coordinates of the displaced control point i, j.

In their initial, undisplaced position, the control points form a rectangular grid:

$$\mathbf{P}_{i,j} = \left(X_{min} + \frac{i}{m}(X_{max} - X_{min}), Y_{min} + \frac{j}{n}(Y_{max} - Y_{min}) \right) \qquad (14.3)$$

as shown in Figure 14.5.

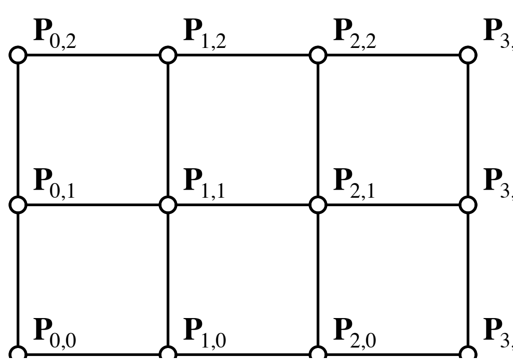

Figure 14.5: FFD undisplaced control points

To compute the location of a point which undergoes deformation, first compute its (s, t) coordinates using equation 14.2. If they are within the range $[0, 1]$, then the point is repositioned using equation 14.1; otherwise, it is not moved.